"Structural vs. Reduced Form"

Language, Confusion, and Models in Empirical Economics

Phil Haile

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Types of Empirical Economics?

1. Measurement
   - how much do prices change with the number of competitors?
   - how has productivity (e.g., TFP) in the U.S. auto industry changed over the last 30 years?
   - what is the effect of college attendance on expected wage?
   - what is the elasticity of aggreg demand for health insurance?
2. Model Testing
   - Is there moral hazard in auto insurance?
   - Does BNE do well in predicting bidding at oil auctions?
   - Do actual contracts resemble optimal contracts?
3. Model Estimation for Counterfactuals
   - How much would prices rise two firms merged? if the sales tax increased by 1%?
   - How would student outcomes differ under a different school choice mechanism?

Measurement vs. "Model Estimation for Counterfactuals"

This distinction is *usually* false: Most "measurement" questions in economics concern a counterfactual of the form "how would the world have been different if XXX were changed, all else fixed?"

- *effect of entry on prices:* we can't just compare markets with differing numbers of competitors
- *productivity*: we can't just look at inputs and outputs to learn the production function (and thus TFP), since unobservables affect both the input levels chosen by firms and their realized output
- *returns to college*: we can't just compare wages for those who attended and those who did not
- *elasticity of demand*: we can't just compare quantities purchased for goods with high prices vs. those with low prices.

Causality and Counterfactuals

Much of empirical economics is about "causal effects"
(what we once just called "effects")

Causality is a concept defined by a counterfactual question of the form "what would happen if certain things were changed while others were held fixed?"

This is true even for an RCT. There, after defining the counterfactual question of interest, one directly creates the counterfactual world—e.g., assigning drugs randomly instead through the mix of decisions that would otherwise prevail. Sometimes the counterfactual question of interest is simple; but only after defining the question can one ask how to design the RCT to answer it.

Types of Empirical Work: A Proper Taxonomy

1. **Descriptive**: estimate relationships between observables
   - establish facts about the data, e.g.,
     - college grads earn 98% more per hour than others
     - income inequality higher now than 30 years ago
     - health care costs growing more slowly after ACA
     - airline prices higher now than before merger wave
   - facts sometimes suggest causal relationships
2. **Structural:** estimate features of a data generating process (i.e., a model) that are (assumed to be) invariant to the policy changes or other *counterfactuals* of interest
   - estimate demand for schools→predict outcomes under a voucher system
   - estimate model of schooling, marriage, and labor supply choices→measure specific notions of the male-female wage gap
   - estimate demand and firm costs→predict the welfare effects of a merger.

What about Program Evaluation?

Consider a RCT with interest only in the effect of the treatment in the population studied.

Here, program evaluation is *descriptive:* it involves only a characterization of relationships between observables.

It is also *structural*: it quantifies features of the underlying ("causal"/"structural") model characterized by the joint distribution

$$F(Y, D, X)$$

where $D \in \{0, 1\}$ indicates treatment, Y measures outcome, X are "controls."

Program Evaluation, Counterfactuals, Models

Program evaluation (indeed, any type of "causal inference") is always a form of structural estimation. It requires a set of maintained hypotheses about the world (i.e., a model) allowing one to *define* and *identify* a counterfactual quantity of interest.

TT, ATE, LATE, QTE, etc. are all precisely defined only under a well specified model of how the data are being generated. Any suggestion that these objects are "model free" is nonsense.

Program Evaluation is Structural Estimation

Sometimes (e.g., in RCT) data description directly reveals a causal/structural/countefactual quantity like TT. But that is a result—one that follows from an explicit set of assumptions on the underlying structure that allows us to distinguish the notion of TT from others and to logically conclude that the data directly reveal this quantity.

Typically program evaluation requires more than descriptive analysis: one must counterfactually hold all else equal to learn the true effect(s) of D on Y, given X. This means treating $F(Y, D, X)$ (or a functional of F like LATE) as the counterfactual quantity of interest and using appropriate econometric techniques (IV, diff-in-diff, RD,...) to estimate it.

Causal Effects, Causal Inference

Old Wine, New Labels

Causality is an abstract notion defined by a counterfactual (see, e.g., Pearl).

The term "causal inference" is typically used to signal restriction to a small class of models (e.g., Rubin causal model) and estimation methods. One can certainly have preferences over models and methods.

But causal estimation/inference is always a special case of structural estimation/inference as defined in econometrics 70+ years ago.

Language and Science

It is great that more economists are interested in empirical work. Great that new techniques have been developed. Great that the standards for convincing ("credible") empirical work have risen.

Sometimes "re-branding" of an old product (e.g., effect$\rightarrow$causal effect) is useful marketing that stimulated new work. But relabeling doesn't change anything on its own. Unfortunately, many (even very accomplished) people are misled by re-labeling of established concepts or alternative uses of established terminology. This is bad for science.

A Final Observation about Counterfactuals

A counterfactual may involve a feasible policy change (e.g., raise the minimum wage). But it could instead be a purely artificial notion (e.g., change women to men to see the resulting wage change).

Obviously one must think carefully about what the artificial ones mean: e.g., woman switched to man at age 25? at birth? But those suggesting that one cannot define the (causal) effect of being female rather than male are confused. A "hypothetical policy intervention" is often a good way to think about a counterfactual, but it is an illustration of the concept, not its definition.

What about "Reduced Form"?

Definition. A **reduced form** is a functional or stochastic mapping for which the inputs are (i) *exogenous* variables and (ii) unobservables ("structural errors"), and for which the outputs are *endogenous* variables. e.g., $Y = f(X, Z, U)$.

This is the textbook definition and the only formal definition I am aware of. In econometrics this goes back at least to 1950.

Where does it come from?

Formally, a reduced form is obtained by solving a (structural) model for each endogenous variable as a function of the exogenous observables and structural errors.

The classic example is perfectly competitive supply and demand:

$$\begin{aligned} Q &= D\left(P, X, U_d\right) \qquad \text{(demand)} \\ P &= MC\left(Q, Z, U_s\right) \qquad \text{(supply)} \end{aligned}$$

Solving for equilibrium yields the reduced form relations

$$\begin{aligned} P &= p\left(Z, X, U_s, U_d\right) \\ Q &= q\left(Z, X, U_s, U_d\right). \end{aligned}$$

Reduced Form: One Source of Confusion

Solving for the reduced form isn't actually essential. We just need to know what goes on which side and what restrictions, if any, must be imposed. Logically one can hypothesize a reduced form rather than hypothesizing structural relationships and deriving the reduced form.

Reduced Form: Another Source of Nuance and Confusion

Definition. A **reduced form** is a functional or stochastic mapping for which the inputs are (i) *exogenous* variables and (ii) unobservables ("structural errors"), and for which the outputs are *endogenous* variables. e.g., $Y = f(X, Z, U)$.

But what does *exogenous* mean?

- for a theorist, an exogenous variable is one taken as given in (not determined within) the model
- for an econometrician: an exogenous variable is one satisfying some kind of independence condition with respect to unobservables.

These can be different: the fact that something is treated as "given" does not mean it satisfies any independence conditions!

An Example: Monopoly Price and Quantity

- firm has marginal cost $c(q, t, \epsilon)$ where q is quantity, t is quality, and ϵ is a cost shock (unobserved shifter of costs)
- demand $D(p, t, \eta)$, where p is price, η is a demand shock
- taking (t, ϵ, η) as given, solve for eqm price and quantity:

$$\begin{aligned} & p^*(t, \epsilon, \eta) \\ & q^*(t, \epsilon, \eta) = D(p^*(t, \epsilon, \eta), t, \eta) \end{aligned}$$

The theorist's reduced forms are $p^*(t, \epsilon, \eta)$ and $q^*(t, \epsilon, \eta)$.

From the econometrician's perspective, these may not be reduced forms because t may be correlated with (ϵ, η). For example, correlation is generally *implied* if t is chosen by the firm with knowledge of (ϵ, η). In that case, t cannot appear on the RHS of a reduced form econometric model.

Example continued

So there is a nuance. But the real issue in the example is that the theorist's model left out something: how t was determined.

That may be fine for the purpose of the theory, but may mean that the econometrician has to *develop a richer model* in which more of the variables are endogenous. This is *always*, at least implicitly, what one is doing when discussing problems of "confounding factors"/selection/endogeneity/IV, etc.

Reduced Form: Some Observations

1. RF requires at least implicit reference to a (structural) model: one cannot know what the arguments of the reduced form are without at least having some notion of a structural model in mind
2. RF equation can involve a fully flexible function, an approximation (e.g., linear) to fully flexible function, or can take a fctl form determined by solving the (parametric or nonparametric) structural model
3. RF can be used for some types of counterfactuals! (those that do not change the mapping from exogenous variables and structural errors to endogenous outcomes)
4. Sometimes there is no difference between the structural equation and reduced form: e.g., exogenous treatment with $Y = f(X, U)$ and scalar $U \perp\!\!\!\perp X$.

“Reduced Form”

Confusion and Abuse of Terminology 101

“Reduced form” is sometimes used to mean “equation I won’t derive, justify, or take questions on, but which I will nonetheless treat as structural (i.e., ‘causal’) when I talk about conclusions”

This is just bad science. There is, of course, bad science of every flavor.

"Reduced Form"

Confusion and Abuse of Terminology 102

"Reduced Form" label often combined with use of IV, RD, etc. due to endogeneity/selection: e.g., "reduced-form demand model" or "reduced-form estimation of the LATE"

For this to make sense, one must mean that the model estiamted is "structural" in the econometrics sense but "reduced form" in yet another sense: one has collapsed a more complex model into a simpler representation, e.g., by-passing some underlying mechanisms. This would be a ""reduced form structural model"."

But EVERY model involves collapsing a more complex world into to simpler representation! There is no coherent way to label only some models as reduced form in this alternative sense. A label that applies to everything has no meaning, yet somehow "reduced form" is viewed as an important term separating "us" from "them."

“Reduced Form”

Confusion and Abuse of Terminology 103

Sometimes reduced-form terminology used with the correct intent but without enough attention to economics to avoid error.

Suppose someone has in mind the supply and demand model

$$\begin{aligned} Q &= D(P, X, U_d) \qquad \text{(demand)} \\ P &= MC(Q, Z, U_s) \qquad \text{(supply)} \end{aligned}$$

but then posits a reduced-form pricing equation (“first-stage”) of the form:

$$P = g(Z, X, \epsilon) \qquad \epsilon \in \mathbb{R}$$

This pricing equation is consistent with the supply and demand model *only* if the two original structural errors U_d and U_s enter the equilibrium solution through a scalar index $\epsilon(U_d, U_s)$: this is a strong functional form requirement (even if g is unrestricted).

Things only get worse if substitutes or complements exist!

Name Your Structural Errors!

The supply and demand example illustrates a general substantive point: it is important to ask what the unobservable(s) are in the relevant economic model, rather than treating them as unnamed "residuals" or "error terms." By asking what ϵ above is, one would realize that it must reflect both demand shocks and cost shocks.

It is hard to speak coherently about the properties of unnamed unobservables!

Reduced Form vs. Descriptive

Baby and Bathwater

"Reduced-form" sometimes used to mean descriptive, sometimes to mean that the structural model is viewed as simple, sometimes to mean "sloppy," and sometimes in a way consistent with its formal definition.

Many interesting papers involve descriptive analysis that evaluates model predictions or suggests patterns/phenomena that one might investigate further using other methods. And for many questions, simple structural models make sense and allow one to answer questions of interest. Mis-labeling these things as "reduced-form" causes confusion and guilt by association with sloppy work.

Reduced Form: The Bottom Line

Most uses of "reduced form" in economics today are either wrong or logically incoherent.

At a minimum this fuzzes communication and confuses many people. Economics would be better if we used this term only when we really mean a relation with endogenous variables on the left, exogenous variables and structural errors on the right.

“Structural”

Confusion and Abuse of Terminology 201

Often “structural” mis-used…

- to describe *how* one estimates rather than *what* one estimates: “We *structurally estimate* a model of...”
  - this is nonsense
- that one is estimating the “deepest” primitives one thinks of
  - this defines terminology based on the speaker’s knowledge/imagination. It is also just incorrect
  - (e.g., don’t expect sub-atomic particles to come up in empirical economics except when someone wants to explain why a problem is too high-dimensional to be tractable).

“Structural”

Confusion and Abuse of Terminology 202

Sometimes “structural” mis-used…

- to mean use of a complex model, or a model with many parametric/functional form assumptions
    - this is completely orthogonal to the question of structural vs. descriptive vs reduced form;
- to mean that all mechanisms have been modeled (again, nonsense)
- to mean that all functional forms and distributions have been specified up to a finite parameter vector
    - this is wrong; there is a big literature on identification of nonparametric structural models

Structural: The Bottom Line

The misuses of "structural," like those of "reduced form," come from both sides of the artificial divide. This results in more more confusion and more baby going out with bathwater.

Causal inference is a special case of structural estimation. One should debate the merits of all forms of structural estimation, but there is no label that accurately separates the good from the bad. Recognize this and you will cut through a great deal of confusion that currently limits communication and progress in economics.

“Structural”

Some Observations (Clear already in the 40s and 50s)

Structural estimation can involve estimation of “deep primitives” like distributions of consumer preferences over product characteristics, “less deep primitives” like aggregate demand elasticities, or a particular function of the primitives like the change in welfare under a particular counterfactual.

For example, we don’t need to know entire demand curve to measure change in consumer surplus due to a small price change: local properties of the demand curve are enough.

(See “sufficient statistics” approach: this is just a standard type of structural econometrics already used for decades. This is old wine in new bottles + nonsensical claim that this is not structural or merely “quasi-structural.”)

Structural"

Some Observations (also written down already in the 40s and 50s)

Although we think of "structure" as a set of policy-invariant features of a model, exactly what "policy invariant" means may vary with the kinds of policy or questions we consider. An example is short-run vs. long-run demand elasticities, which reflect responses to two different counterfactuals.

Structural estimation/inference can involve estimation/inference on points or sets (e.g., bounds on LATE or on a demand curve).

Identification

The concept of identification has gotten much wider attention in recent decades than in the past. This is good: it reflects greater attention to arguably the fundamental challenge of empirical economics: whether/how the things we observe are capable of revealing the answers to the questions we care about.

Identification Defined Formally

(see e.g., Hurwicz (1950), Koopmans and Reiersol (1950), Berry and Haile (2018))

- a structure S is a data generating process, i.e., a set of probabilistic or functional relationships between the observable and latent variables that implies ("generates") a joint distribution of the observables
- let $\mathfrak{S}$ = the set of all structures; $S_0 \in \mathfrak{S}$ the true structure
- a hypothesis is any nonempty subset of $\mathfrak{S}$
- hypothesis $\mathcal{H}$ is true (satisfied) if $S_0 \in \mathcal{H}$
- a structural feature $\theta(S_0)$ is a functional of the true structure

Definition. A structural feature $\theta(S_0)$ is identified (or *point identified*, or *identifiable*) under the hypothesis $\mathcal{H}$ if $\theta(S_0)$ is uniquely determined within the set $\{\theta(S) : S \in \mathcal{H}\}$ by the joint distribution of observables.

Remarks on Identification

- *Identification cannot be defined without the notion of a true structure within a class defined by maintained hypotheses (what we usually call a "model").* The model may be simple or complicated, may involve economics or only hypothesized statistical relationships (e.g., Rubin causal model). But the very concept of identification presumes that there are structural features—abstract notions—that one wishes to uncover.
- *Identification has nothing to do with a given sample or an estimator.* In fact, strictly speaking it is not even about what one could learn from an infinitely large sample
- *Identification of a structural feature $\theta(S_0)$ may hold even when the true structure S_0 is itself not identified.* An example is the reduced form (note: identification of the reduced from is not automatic–see supply and demand!).

Does all this language really matter?

Or is this only Phil's pet peeve?

"Oxford English Dictionary View": A word's everyday use determines its meaning, not the other way around. It is nonsense to assert that a word's common use is incorrect.

My View: The Words Economists Use Matter

(1) Misuse/inconsistent use of terminology destroys information. This may be mildly annoying or amusing in everyday conversation

> *"I literally died laughing when I heard what Josh Angrist said about empirical IO!"*

but is sloppiness that should be avoided in serious scholarship.

(2) The language we use shapes the way we think and how we are understood. Precise language encourages precise thinking and transparent exchange of ideas. This is part of our job.

(3) We should debate modeling approaches and tradeoffs between the assumptions relied upon and the questions answered. This is what "structural vs. reduced form" divide is really about—i.e., different types of structural models. Misuses of language creates a false barrier that shuts down those conversations, muddies the waters, and slows scientific progress.

The Role of Economic Models in Empirical Work

A Case for Structure Based on Economic Models

We are economists, not statisticians.

- Statisticians are good at describing the data.
- Economists are good at interpreting it using formal logic:

  given a set of maintained hypotheses, the data imply . . .

- Where do the maintained hypotheses come from? How can they be evaluated? How do we know which maintained hypotheses are useful?

Economic Models in Empirical Work

Many important questions can be answered only by exploiting economic models (vs. statistical or DAG models) to provide a logical framework for interpreting the data:

- to tell us what to look at: what are the structural features of interest for our questions? One should not define the object of interest based only on what some statistical procedure produces!
- to define what it means to have a "valid" estimation method
- to provide functional/probabilistic relationships that can be used to estimate of the structural features of interest; e.g.,
    - optimality conditions that relate observables to primitives
    - IV conditions (absent an experiment, what are valid instruments? this requires economic reasoning, which means at least an informal model).

The Role of Economic Models, An Example: Part 1

A researcher observes price P and quantity Q of a good in many markets

- He says: "I do not want to impose arbitrary restrictions from a model. I just want to let the data speak."
- He regresses Q on P, finds a positive correlation, and concludes "Initial evidence suggests that P has a positive effect on Q."
- He then adds some covariates Z to the regression and obtains similar results. The researcher concludes, "The positive effect of P on Q is robust to the inclusion of a rich set of controls."

(So far, nonsense.)

An Example: Part 2

- The researcher now considers the use of instrumental variables methods, characterizing this either as a robustness check on the original analysis, or as a way of controlling for possible "confounds"
- The researcher suggests that a measure of the availability of a substitute goods be used as an instrument for P. TSLS now reveals a stronger positive "causal effect" of P on Q. The researcher concludes that the original results are qualitatively robust, but that controlling for endogeneity of P eliminates a downward bias in the OLS estimates.

(still nonsense)

An Example: Part 3

- Another researcher reads the paper and has a different idea for an instrument: the manufacturing wage in the local market, something also left out of Z that plausibly affects prices.
- TSLS now yields a precisely estimated *negative* "causal effect" of price on quantity
- The researcher concludes, "the causal effect of P on Q is heterogeneous. The effect one measures depends on which prices are changing in response to variation in the instrument one uses."

(more nonsense).

Why Do We Know This Is Nonsense?

Our researcher started with a common approach: he (correctly!) conjectures that a variable X may affect another, Y, and explores the relationship with regression, with or without IV, interpreting at least the latter as "the causal effect of X on Y." Where did he go wrong? How do *we know* that something has gone wrong?

Why Do We Know This Is Nonsense?

Here we know something is wrong because we already have a deeply ingrained idea of how prices and quantities are determined in a market. We automatically have at least elements of a structural model in mind.

However, your intuition may not be good enough. If one writes down a model of demand allowing for the existence of substitute goods, one makes a discovery: just as prices of substitutes affect demand, so must any unobservables associated with substitutes! If one wants to learn about demand, a regression approach is generally not even valid!

Thus, even when we have a strong intuition, failing to write down the economic model that justifies your empirical approach will often lead to an error. Many published papers (even by very famous people) make exactly this error when considering demand estimation!

Avoiding Nonsense

Once the researcher suspects that there may be omitted factors, selection, endogeneity, etc., *some type of model is necessary* to determine what "fixes" will work (indeed, what "work" even means). In the example a model is needed...

- to recognize demand and supply as distinct objects—that *there is no such thing as "the causal effect of P on Q"*
- to define what objects should be measured
- to recognize which structural errors must be dealt with (e.g., held fixed to measure a ceterius paribus response of demand to price)
- to define what it would mean to have a valid estimation approach or valid IV (and these may appear when one writes down a model).

The Role of Models

Stepping Back

"all models are wrong, but some are useful." – George Box

Useful at a minimum because without a model of some kind there is typically only hand waving. Attempts to go beyond data description without a model are "not even wrong" — i.e., one cannot even define what "right" means.

The Role of Models

Stepping Back

"Art is not truth. Art is a lie that makes us realize truth..." – Pablo Picasso

The art of empirical work includes selecting a model that captures essential features *for the purpose at hand* and allows one to justify an interpretation of a measurement. This will involve assumptions that one could question, debate, reject, or improve upon.

But only by specifying a model can one speak coherently about whether the maintained assumptions are problematic, whether certain data allow measurement, what alternative assumptions might imply, and how science might progress.

Summary

1. Be careful about the language of empirical economics. Don't be fooled by common (ubiquitous!) abuses. Try not to become part of the problem yourself. Better yet, look for opportunities to overcome the false barriers by bringing insights from artificially disconnected literatures together.
2. Don't underestimate the extent to which an economic model can be useful—even essential—to good empirical work.

   My part of the course will illustrate some roles models can play.

Fun Reading

Koopmans, T. (1947). “Measurement Without Theory,” Cowles Foundation Discussion paper 25a. http://www.jstor.org/stable/1928627

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Marschak, J. and W.C. Andrews Jr. (1944), “Random Simultaneous Equations and the Theory of Production,” *Econometrica*, 12, 143-205., http://www.jstor.org/stable/1905432